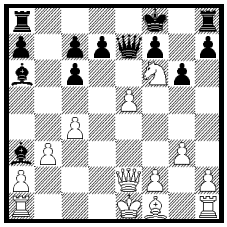
# Appendix A

# The chess middle-game test set

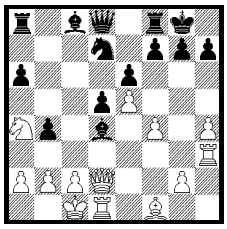
For the transposition-table experiments on chess middle-game positions described in Chapter 2 the following test set has been used.

- The 6 WTM positions from move 15 onwards of the following game:
  **Kasparov–Ivanchuk, Amsterdam (round 1) 1994**
  1. e4 e5 2. ♘f3 ♘c6 3. d4 exd4 4. ♘xd4 ♘f6 5. ♘xc6 bxc6 6. e5 ♕e7 7. ♕e2 ♘d5 8. c4 ♗a6 9. b3 g6 10. ♗a3 ♕g5 11. g3 ♘c3 12. ♘xc3 ♗xa3 13. ♘e4 ♕e7 14. ♘f6+ ♔f8 15. ♗g2 ♗b4+ 16. ♔f1 ♖d8 17. ♕b2 ♗a3 18. ♕c3 ♗b4 19. ♕b2 ♗a3 20. ♕c3 ♗b4 $\frac{1}{2}$–$\frac{1}{2}$



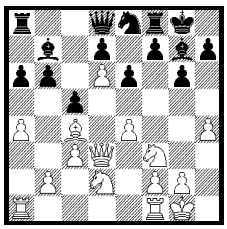
Move 15 (WTM)

- The 18 WTM positions from move 15 onwards of the following game:
  **Kasparov–Short, Amsterdam (round 2) 1994**
  1. e4 e6 2. d4 d5 3. ♘c3 ♘f6 4. e5 ♘fd7 5. f4 c5 6. ♘f3 ♘c6 7. ♗e3 cxd4 8. ♘xd4 ♗c5 9. ♕d2 0–0 10. 0–0–0 a6 11. h4 ♘xd4 12. ♗xd4 b5 13. ♖h3 b4 14. ♘a4 ♗xd4 15. ♕xd4 f6 16. ♕xb4 fxe5 17. ♕d6 ♕f6 18. f5 ♕h6+ 19. ♔b1 ♖xf5 20. ♖f3 ♖xf3 21. gxf3 ♕f6 22. ♗h3 ♔f7 23. c4 dxc4 24. ♘c3 ♕e7 25. ♕c6 ♖b8 26. ♘e4 ♘b6 27. ♘g5+ ♔g8 28. ♕e4 g6 29. ♕xe5 ♖b7 30. ♖d6 c3 31. ♗xe6+ ♗xe6 32. ♖xe6 1–0



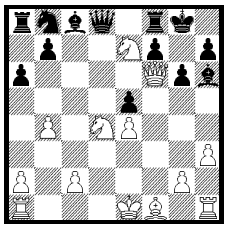
Move 15 (WTM)

- The 20 BTM positions from move 15 up to and including move 34 of the following game:
  **Timman–Kasparov, Amsterdam (round 3) 1994**
  1. d4 ♘f6 2. ♘f3 g6 3. ♗g5 ♗g7 4. c3 b6 5. ♗xf6 ♗xf6 6. e4 ♗b7 7. ♗d3 c5 8. d5 e6 9. ♗c4 0–0 10. 0–0 ♘a6 11. ♕d3 ♘c7 12. d6 ♘e8 13. ♘bd2 ♗g7 14. h4 a6 15. a4 ♕b8 16. e5 f6 17. h5 fxe5 18. hxg6 h6 19. ♖fe1 ♕xd6 20. ♕xd6 ♘xd6 21. ♘xe5 ♗xe5 22. ♖xe5 ♖f4 23. ♗d3 ♖af8 24. f3 a5 25. ♔f2 ♔g7 26. ♖h5 ♘e8 27. ♔g3 ♘f6 28. ♖e5 ♘d5 29. ♗e4 ♖4f6 30. ♘c4 ♘f4 31. ♗xb7 ♖xg6+ 32. ♔h2 ♖xg2+ 33. ♔h1 d5 34. ♘xb6 ♖b8 35. ♖xe6 ♖xb7 36. ♖d6 ♖g5 37. ♖d1 d4 38. ♘c4 ♔h7 39. ♖e1 ♖h5+ 40. ♔g1 ♖g7+ 0–1



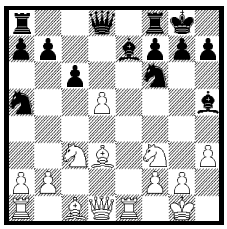
Move 15 (BTM)

- The 24 BTM positions from move 15 onwards of the following game:
  **Ivanchuk–Kasparov, Amsterdam (round 4) 1994**
  1. e4 c5 2. ♘f3 d6 3. d4 cxd4 4. ♘xd4 ♘f6 5. ♘c3 a6 6. f4 ♕c7 7. ♕f3 g6 8. ♗e3 ♗g7 9. h3 e5 10. fxe5 dxe5 11. ♗h6 ♗xh6 12. ♕xf6 0–0 13. ♘d5 ♕a5 14. b4 ♕d8 15. ♘e7+ ♕xe7 16. ♕xe7 exd4 17. ♗c4 ♘c6 18. ♕c5 ♗e3 19. ♖f1 ♘d8 20. ♖f3 ♗e6 21. ♖xe3 dxe3 22. ♗xe6 ♘xe6 23. ♕xe3 a5 24. b5 ♖ac8 25. 0–0–0 ♖c5 26. ♖d5 b6 27. ♕g3 ♖c7 28. ♕d6 ♖fc8 29. ♖d2 ♖b7 30. g4 ♘c5 31. ♕f6 h6 32. e5 ♖e8 33. h4 ♔h7 34. h5 g5 35. ♖d6 ♖e6 36. ♕d8 ♔g7 37. a3 a4 38. ♔b2 ♖be7 39. ♖xb6 1–0



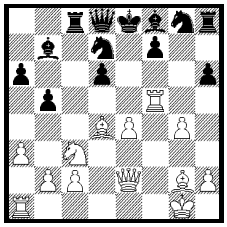
Move 15 (BTM)

- The 15 WTM positions from move 15 up to and including move 29 of the following game:
  **Kasparov–Timman, Amsterdam (round 5) 1994**
  1. e4 e5 2. ♘f3 ♘f6 3. ♘xe5 d6 4. ♘f3 ♘xe4 5. d4 d5 6. ♗d3 ♘c6 7. 0–0 ♗e7 8. ♖e1 ♗g4 9. c4 ♘f6 10. ♘c3 dxc4 11. ♗xc4 0–0 12. d5 ♘a5 13. ♗d3 c6 14. h3 ♗h5 15. ♖e5 ♗g6 16. ♗g5 ♗d6 17. ♖e2 ♗b4 18. ♗xf6 gxf6 19. ♖c1 ♖c8 20. ♘e4 f5 21. ♘g3 ♕xd5 22. a3 ♗d6 23. ♘xf5 ♖cd8 24. ♖e5 ♗xe5 25. ♘e7+ ♔g7 26. ♘xd5 ♗xb2 27. ♘f4 ♗xd3 28. ♘xd3 ♗xc1 29. ♕xc1 ♖xd3 30. ♕g5+ 1–0



Move 15 (WTM)

- The 11 BTM positions from move 15 onwards of the following game:
  **Short–Kasparov, Amsterdam (round 6) 1994**
  1. e4 c5 2. ♘c3 e6 3. ♘f3 a6 4. d4 cxd4 5. ♘xd4 d6 6. g4 b5 7. a3 h6 8. ♗g2 ♗b7 9. 0–0 ♘d7 10. f4 ♖c8 11. ♗e3 g5 12. ♕e2 gxf4 13. ♖xf4 e5 14. ♖f5 exd4 15. ♗xd4 ♘e5 16. ♘d5 ♗g7 17. ♖af1 ♖h7 18. ♔h1 ♗h8 19. c3 ♘e7 20. ♗xe5 dxe5 21. ♕f3 ♘xf5 22. ♕xf5 ♖g7 23. ♘f6+ ♔f8 24. ♘d7+ ♔g8 25. ♘f6+ ♔f8 $\frac{1}{2}$–$\frac{1}{2}$



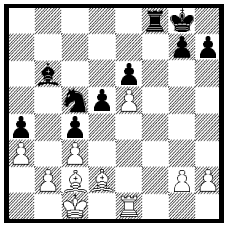
Move 15 (BTM)

# Appendix B

# The chess endgame test set

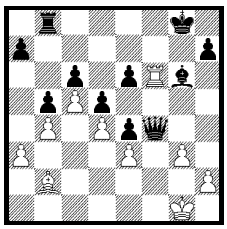
For the transposition-table experiments on chess endgame positions described in Chapter 2 the following test set has been used.

- The 28 WTM positions from move 31 onwards of the following game:
  **Gossip–Mason, New York (round 20) 1889**
  1. e4 e6 2. d4 d5 3. ♘c3 ♘f6 4. e5 ♘fd7 5. f4 c5 6. dxc5 ♘c6 7. ♘f3 ♗xc5 8. ♘e2 ♕b6 9. c3 ♗f2+ 10. ♔d2 ♕e3+ 11. ♔c2 ♕e4+ 12. ♕d3 ♘c5 13. ♕xe4 ♘xe4 14. ♘ed4 ♗d7 15. ♘xc6 bxc6 16. ♗d3 ♘c5 17. ♗e2 ♘b7 18. ♖f1 ♗b6 19. ♗d2 c5 20. ♗a6 ♖b8 21. ♖ae1 ♘d8 22. ♗d3 a5 23. ♔c1 a4 24. a3 ♘b7 25. f5 c4 26. fxe6 ♗xe6 27. ♗c2 ♘c5 28. ♘d4 0–0 29. ♘xe6 fxe6 30. ♖xf8+ ♖xf8 31. ♗e3 ♘b3+ 32. ♗xb3 ♗xe3 33. ♖xe3 cxb3 34. ♖e2 ♖f4 35. ♔d2 ♔f7 36. ♖e3 ♖f2+ 37. ♖e2 ♖xe2+ 38. ♔xe2 ♔g6 39. ♔e3 ♔f5 40. ♔d4 h5 41. g3 g5 42. h3 h4 43. g4+ ♔f4 44. ♔c5 ♔xe5 45. ♔b4 d4 46. ♔xa4 d3 47. ♔xb3 ♔e4 48. a4 ♔e3 49. a5 d2 50. ♔b4 d1♕ 51. b3 ♕a1 52. c4 ♔d4 53. ♔b5 ♕c3 54. ♔c6 ♕xb3 55. a6 ♕xc4+ 56. ♔b7 ♕b5+ 57. ♔a7 ♔c5 58. ♔a8 ♔c6 0–1



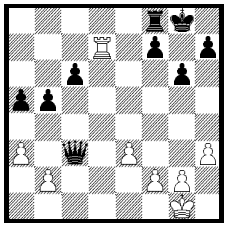
Move 31 (WTM)

- The 21 WTM positions from move 34 onwards of the following game:
  **Rabinovich–Romanovsky, Leningrad 1934**
  1. c4 ♘f6 2. ♘c3 c6 3. d4 d5 4. ♘f3 ♘e4 5. e3 e6 6. ♗d3 f5 7. ♕c2 ♘d7 8. b3 ♗b4 9. ♗b2 ♕a5 10. ♖c1 0–0 11. 0–0 ♗d6 12. ♘e2 ♕d8 13. ♘e5 ♕h4 14. f3 ♘ec5 15. g3 ♕h6 16. ♘f4 ♘xd3 17. ♘exd3 g5 18. ♘g2 ♘f6 19. ♖ce1 g4 20. fxg4 ♘xg4 21. ♘gf4 ♘f6 22. ♖e2 ♖f7 23. b4 ♘e4 24. ♘c5 ♖b8 25. a3 b6 26. ♘xe4 fxe4 27. ♖ef2 ♗d7 28. c5 ♗xf4 29. ♖xf4 ♖xf4 30. ♖xf4 b5 31. ♕f2 ♗e8 32. ♖f6 ♗g6 33. ♕f4 ♕xf4 34. ♖xf4 h5 35. h3 ♔g7 36. ♗c3 ♗f5 37. g4 hxg4 38. hxg4 ♗g6 39. ♔g2 ♔h6 40. ♖f6 ♖e8 41. ♗e1 ♔g7 42. ♖f1 ♖a8 43. ♔h3 a6 44. ♗g3 ♖h8 45. ♗h4 ♖f8 46. ♖xf8 ♔xf8 47. ♗g3 e5 48. ♗xe5 ♔f7 49. ♔h4 ♔e6 50. ♔g5 ♗e8 51. ♔h6 ♗f7 52. ♔g7 ♗e8 53. g5 ♔f5 54. ♔f8 1–0



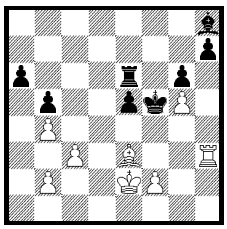
Move 34 (WTM)

- The 17 WTM positions from move 26 onwards of the following game:
  **Capablanca–Alekhine, Buenos Aires World Championship (game 5) 1927**
  1. d4 d5 2. c4 e6 3. ♘c3 ♘f6 4. ♗g5 ♘bd7 5. e3 c6 6. a3 ♗e7 7. ♘f3 0–0 8. ♗d3 dxc4 9. ♗xc4 ♘d5 10. ♗xe7 ♕xe7 11. ♖c1 ♘xc3 12. ♖xc3 e5 13. dxe5 ♘xe5 14. ♘xe5 ♕xe5 15. 0–0 ♗e6 16. ♗xe6 ♕xe6 17. ♖d3 ♕f6 18. ♕b3 ♕e7 19. ♖fd1 ♖ad8 20. h3 ♖xd3 21. ♖xd3 g6 22. ♕d1 ♕e5 23. ♕d2 a5 24. ♖d7 b5 25. ♕c3 ♕xc3 26. bxc3 ♖c8 27. ♔f1 ♔g7 28. ♖a7 a4 29. c4 ♔f6 30. ♖a5 ♔e6 31. ♔e2 bxc4 32. ♖c5 ♔d6 33. ♖xc4 ♖a8 34. ♖d4+ ♔e6 35. ♔d3 c5 36. ♖h4 h5 37. g4 hxg4 38. ♖xg4 ♔d6 39. ♖f4 f5 40. ♖h4 ♔d5 41. ♔c2 ♖a6 42. ♔c3 $\frac{1}{2}$–$\frac{1}{2}$



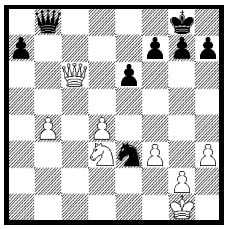
Move 26 (WTM)

- The 17 WTM positions from move 38 onwards of the following game:
  **Fischer–Reshevsky, New York US Championship (round 5) 1962**
  1. e4 c5 2. ♘f3 d6 3. d4 cxd4 4. ♘xd4 ♘f6 5. ♘c3 a6 6. h3 g6 7. g4 ♗g7 8. g5 ♘h5 9. ♗e2 e5 10. ♘b3 ♘f4 11. ♘d5 ♘xd5 12. ♕xd5 ♘c6 13. ♗g4 ♗xg4 14. hxg4 ♕c8 15. ♕d1 ♘d4 16. c3 ♘xb3 17. axb3 ♕e6 18. ♖a5 f6 19. ♕d5 ♕xd5 20. ♖xd5 ♔d7 21. gxf6 ♗xf6 22. g5 ♗e7 23. ♔e2 ♖af8 24. ♗e3 ♖c8 25. b4 b5 26. ♖dd1 ♔e6 27. ♖a1 ♖c6 28. ♖h3 ♗f8 29. ♖h1 ♖c7 30. ♖h4 d5 31. ♖a1 ♖c6 32. exd5+ ♔xd5 33. ♖d1+ ♔e6 34. ♖d8 ♔f5 35. ♖a8 ♖e6 36. ♖h3 ♗g7 37. ♖xh8 ♗xh8 38. ♖xh7 ♖e8 39. ♖f7+ ♔g4 40. f3+ ♔g3 41. ♔d3 e4+ 42. fxe4 ♖d8+ 43. ♗d4 ♔g4 44. ♖f1 ♗e5 45. ♔e3 ♗c7 46. ♖g1+ ♔h5 47. ♔f3 ♖d7 48. e5 ♖f7+ 49. ♔e4 ♖f5 50. e6 ♗d8 51. ♗f6 ♗xf6 52. gxf6 ♖xf6 53. ♔e5 ♖f2 54. ♖e1 1–0



Move 38 (WTM)

- The 29 WTM positions from move 38 onwards of the following game:
  **Lisitsin–Capablanca, Moscow (round 5) 1935**
  1. ♘f3 d5 2. c4 c6 3. e3 ♘f6 4. ♘c3 ♗g4 5. cxd5 ♘xd5 6. ♗e2 e6 7. d4 ♘d7 8. 0–0 ♕c7 9. ♗d2 ♗d6 10. ♘e4 ♘7f6 11. ♘xd6+ ♕xd6 12. ♘e5 ♗xe2 13. ♕xe2 0–0 14. ♖c1 ♘b6 15. ♘d3 ♖e8 16. ♖fe1 ♘bd7 17. h3 ♕d5 18. b3 ♕b5 19. ♗c3 ♘d5 20. ♕d2 ♘xc3 21. ♕xc3 ♖ad8 22. a4 ♕b6 23. b4 ♘f6 24. ♕c4 ♘e4 25. a5 ♕c7 26. a6 ♖c8 27. axb7 ♕xb7 28. ♖a1 ♖c7 29. ♖ec1 ♖b8 30. ♕c2 ♕c8 31. ♖a5 ♖b6 32. ♕a4 ♕b8 33. f3 ♘f6 34. ♖c5 ♘d5 35. ♖xc6 ♖cxc6 36. ♖xc6 ♖xc6 37. ♕xc6 ♘xe3 38. ♘c5 ♘d5 39. b5 ♘b6 40. ♘d7 ♕d8 41. ♘xb6 axb6 42. ♕c4 h5 43. ♔f1 g6 44. ♔g1 ♔g7 45. ♔f1 ♕d6 46. ♔g1 ♕f4 47. ♕c3 ♔h7 48. ♔f1 ♕f5 49. ♕c4 ♔g7 50. ♔f2 ♕g5 51. ♕e2 ♔f6 52. ♕b2 ♕d5 53. ♔e3 e5 54. f4 exf4+ 55. ♔xf4 ♔e6 56. h4 f6 57. ♔e3 ♕c4 58. g3 g5 59. hxg5 fxg5 60. ♕h2 ♕b3+ 61. ♔e4 g4 62. ♕e2 ♕xg3 63. ♕c4+ ♔e7 64. ♕c8 ♕f3+ 65. ♔e5 ♕f6+ 66. ♔d5 ♕d6+ 0–1



Move 38 (WTM)

# Appendix C

# The transposition-table results

This appendix presents the results of the three series of experiments given in Chapter 2. The first series of experiments investigates which replacement scheme performs best. The second series of experiments examines which information is more important to store in a transposition-table entry: the best move in a position, or the score of that move. Finally, the third series of experiments investigates the effect of storing an $n$-ply PV ($n = 2..5$) in an entry, instead of only the best move (a 1-ply PV).

### Comparing replacement schemes

The first series of experiments consists of three parts. First, the 3-ply to 8-ply transposition-table results for the seven replacement schemes (TwoBig1, TwoDeep, Big1, BigAll, Deep, New and Old) on chess middle-game positions are listed in Tables C.1 to C.12. The middle-game figures reported are number of nodes searched in thousands. The 3-ply to 7-ply results are listed with eight table sizes (8K, 16K, 32K, 64K, 128K, 256K, 512K and 1024K). These results are the cumulative results of all six games (94 middle-game positions) given in Appendix A. The 8-ply results are listed with four table sizes (16K, 64K, 256K and 1024K). These results are the cumulative results of the first three games (44 middle-game positions) given in Appendix A. For every ply depth two tables are given: one without time stamping and one with it. In the former case the transposition tables are cleared between moves.

| | TwoBig1 | TwoDeep | Big1 | BigAll | Deep | New | Old |
|---|---|---|---|---|---|---|---|
| 8K | 684 | 684 | 686 | 686 | 687 | 687 | 688 |
| 16K | 684 | 684 | 683 | 683 | 684 | 684 | 685 |
| 32K | 684 | 684 | 684 | 684 | 684 | 684 | 685 |
| 64K | 684 | 684 | 684 | 684 | 684 | 684 | 684 |
| 128K | 684 | 684 | 684 | 684 | 684 | 684 | 684 |
| 256K | 684 | 684 | 684 | 684 | 684 | 684 | 684 |
| 512K | 684 | 684 | 684 | 684 | 684 | 684 | 684 |
| 1024K | 684 | 684 | 684 | 684 | 684 | 684 | 684 |

Table C.1: Replacement-scheme results for the chess middle game (without time stamping, 3-ply searches).

| | TwoBig1 | TwoDeep | Big1 | BigAll | Deep | New | Old |
|---|---|---|---|---|---|---|---|
| 8K | 660 | 660 | 660 | 660 | 665 | 665 | 661 |
| 16K | 660 | 660 | 660 | 660 | 660 | 660 | 660 |
| 32K | 660 | 660 | 660 | 660 | 660 | 660 | 660 |
| 64K | 660 | 660 | 660 | 660 | 660 | 660 | 660 |
| 128K | 660 | 660 | 660 | 660 | 660 | 660 | 660 |
| 256K | 660 | 660 | 660 | 660 | 660 | 660 | 660 |
| 512K | 660 | 660 | 660 | 660 | 660 | 660 | 660 |
| 1024K | 660 | 660 | 660 | 660 | 660 | 660 | 660 |

Table C.2: Replacement-scheme results for the chess middle game (with time stamping, 3-ply searches).

| | TwoBig1 | TwoDeep | Big1 | BigAll | Deep | New | Old |
|---|---|---|---|---|---|---|---|
| 8K | 2,737 | 2,774 | 2,790 | 2,800 | 2,816 | 2,789 | 2,814 |
| 16K | 2,757 | 2,751 | 2,808 | 2,807 | 2,780 | 2,784 | 2,788 |
| 32K | 2,758 | 2,756 | 2,754 | 2,753 | 2,791 | 2,778 | 2,765 |
| 64K | 2,758 | 2,758 | 2,735 | 2,735 | 2,763 | 2,755 | 2,776 |
| 128K | 2,757 | 2,757 | 2,763 | 2,763 | 2,749 | 2,766 | 2,756 |
| 256K | 2,757 | 2,757 | 2,745 | 2,745 | 2,734 | 2,730 | 2,776 |
| 512K | 2,757 | 2,757 | 2,766 | 2,766 | 2,761 | 2,761 | 2,782 |
| 1024K | 2,757 | 2,757 | 2,767 | 2,767 | 2,759 | 2,759 | 2,791 |

Table C.3: Replacement-scheme results for the chess middle game (without time stamping, 4-ply searches).

| | TwoBig1 | TwoDeep | Big1 | BigAll | Deep | New | Old |
|---|---|---|---|---|---|---|---|
| 8K | 2,706 | 2,673 | 2,648 | 2,705 | 2,727 | 2,735 | 2,739 |
| 16K | 2,671 | 2,697 | 2,698 | 2,698 | 2,696 | 2,644 | 2,688 |
| 32K | 2,687 | 2,703 | 2,683 | 2,701 | 2,661 | 2,692 | 2,704 |
| 64K | 2,682 | 2,682 | 2,651 | 2,659 | 2,719 | 2,670 | 2,674 |
| 128K | 2,684 | 2,684 | 2,633 | 2,633 | 2,705 | 2,714 | 2,647 |
| 256K | 2,684 | 2,684 | 2,679 | 2,679 | 2,681 | 2,700 | 2,700 |
| 512K | 2,684 | 2,684 | 2,680 | 2,680 | 2,672 | 2,674 | 2,693 |
| 1024K | 2,684 | 2,684 | 2,680 | 2,680 | 2,673 | 2,674 | 2,701 |

Table C.4: Replacement-scheme results for the chess middle game (with time stamping, 4-ply searches).

| | TwoBig1 | TwoDeep | Big1 | BigAll | Deep | New | Old |
|---|---|---|---|---|---|---|---|
| 8K | 8,967 | 9,186 | 9,185 | 9,206 | 9,341 | 9,312 | 9,435 |
| 16K | 8,932 | 9,004 | 9,101 | 9,109 | 9,221 | 9,141 | 9,235 |
| 32K | 8,988 | 8,999 | 9,020 | 9,079 | 9,061 | 9,046 | 9,169 |
| 64K | 9,014 | 9,016 | 9,000 | 9,006 | 9,037 | 9,089 | 9,091 |
| 128K | 8,985 | 8,975 | 8,945 | 8,946 | 9,024 | 9,089 | 8,965 |
| 256K | 8,993 | 8,982 | 9,036 | 9,038 | 9,010 | 8,955 | 9,052 |
| 512K | 8,964 | 8,964 | 9,003 | 9,003 | 8,967 | 8,968 | 9,066 |
| 1024K | 8,964 | 8,964 | 8,939 | 8,939 | 8,983 | 8,987 | 8,986 |

Table C.5: Replacement-scheme results for the chess middle game (without time stamping, 5-ply searches).

| | TwoBig1 | TwoDeep | Big1 | BigAll | Deep | New | Old |
|---|---|---|---|---|---|---|---|
| 8K | 8,892 | 8,815 | 8,912 | 8,854 | 9,093 | 9,065 | 9,160 |
| 16K | 8,755 | 8,806 | 8,798 | 8,833 | 8,841 | 8,842 | 9,005 |
| 32K | 8,741 | 8,796 | 8,844 | 8,835 | 8,922 | 8,829 | 8,744 |
| 64K | 8,724 | 8,718 | 8,721 | 8,767 | 8,779 | 8,750 | 8,893 |
| 128K | 8,755 | 8,744 | 8,793 | 8,855 | 8,763 | 8,776 | 8,801 |
| 256K | 8,736 | 8,745 | 8,802 | 8,801 | 8,765 | 8,723 | 8,739 |
| 512K | 8,732 | 8,732 | 8,743 | 8,798 | 8,703 | 8,690 | 8,836 |
| 1024K | 8,732 | 8,732 | 8,736 | 8,764 | 8,744 | 8,746 | 8,769 |

Table C.6: Replacement-scheme results for the chess middle game (with time stamping, 5-ply searches).

| | TwoBig1 | TwoDeep | Big1 | BigAll | Deep | New | Old |
|---|---|---|---|---|---|---|---|
| 8K | 40,619 | 42,120 | 42,280 | 42,694 | 42,903 | 45,932 | 46,581 |
| 16K | 39,210 | 39,896 | 40,305 | 40,112 | 40,831 | 42,864 | 43,693 |
| 32K | 38,146 | 38,930 | 39,165 | 39,321 | 39,172 | 40,506 | 41,828 |
| 64K | 38,088 | 38,061 | 38,901 | 38,919 | 38,695 | 39,265 | 39,751 |
| 128K | 38,247 | 37,884 | 38,798 | 38,287 | 38,313 | 38,635 | 38,781 |
| 256K | 37,892 | 38,059 | 37,867 | 38,210 | 37,983 | 38,042 | 38,607 |
| 512K | 38,036 | 37,871 | 37,757 | 38,178 | 38,009 | 38,342 | 38,269 |
| 1024K | 37,959 | 37,790 | 37,901 | 37,734 | 38,454 | 37,810 | 38,243 |

Table C.7: Replacement-scheme results for the chess middle game (without time stamping, 6-ply searches).

| | TwoBig1 | TwoDeep | Big1 | BigAll | Deep | New | Old |
|---|---|---|---|---|---|---|---|
| 8K | 40,148 | 41,390 | 41,846 | 41,421 | 42,747 | 45,249 | 46,631 |
| 16K | 38,190 | 39,367 | 39,680 | 39,380 | 40,780 | 42,457 | 43,294 |
| 32K | 37,187 | 37,695 | 38,583 | 38,344 | 38,638 | 40,169 | 40,462 |
| 64K | 37,008 | 36,898 | 37,764 | 37,555 | 37,813 | 38,940 | 38,501 |
| 128K | 36,754 | 36,959 | 37,077 | 37,131 | 37,004 | 37,437 | 37,781 |
| 256K | 36,854 | 36,705 | 36,697 | 37,053 | 36,689 | 38,015 | 37,529 |
| 512K | 36,865 | 36,658 | 36,446 | 36,613 | 36,756 | 36,901 | 36,894 |
| 1024K | 36,914 | 36,425 | 36,983 | 36,265 | 36,462 | 36,690 | 36,447 |

Table C.8: Replacement-scheme results for the chess middle game (with time stamping, 6-ply searches).

| | TwoBig1 | TwoDeep | Big1 | BigAll | Deep | New | Old |
|---|---|---|---|---|---|---|---|
| 8K | 162,393 | 168,136 | 175,713 | 177,681 | 181,147 | 199,937 | 205,747 |
| 16K | 149,363 | 153,613 | 162,211 | 162,578 | 165,565 | 181,221 | 191,766 |
| 32K | 141,105 | 143,399 | 149,651 | 150,810 | 153,938 | 165,531 | 174,633 |
| 64K | 136,147 | 138,046 | 141,101 | 142,081 | 143,767 | 152,896 | 158,519 |
| 128K | 132,571 | 134,148 | 136,419 | 136,996 | 136,879 | 142,206 | 146,668 |
| 256K | 131,739 | 131,720 | 133,513 | 134,047 | 134,900 | 139,362 | 138,438 |
| 512K | 131,511 | 131,092 | 131,961 | 133,667 | 134,183 | 135,244 | 136,112 |
| 1024K | 131,722 | 130,997 | 132,152 | 132,130 | 132,938 | 132,597 | 133,623 |

Table C.9: Replacement-scheme results for the chess middle game (without time stamping, 7-ply searches).

| | TwoBig1 | TwoDeep | Big1 | BigAll | Deep | New | Old |
|---|---|---|---|---|---|---|---|
| 8K | 159,463 | 166,755 | 177,310 | 178,308 | 181,614 | 196,798 | 216,668 |
| 16K | 149,330 | 153,771 | 160,847 | 161,390 | 166,035 | 179,992 | 200,805 |
| 32K | 139,768 | 144,890 | 149,229 | 149,105 | 152,088 | 164,413 | 179,437 |
| 64K | 135,154 | 137,008 | 140,681 | 140,523 | 144,751 | 150,047 | 160,516 |
| 128K | 131,154 | 132,456 | 134,055 | 134,473 | 137,187 | 141,991 | 145,095 |
| 256K | 129,206 | 127,970 | 131,685 | 131,598 | 132,414 | 136,596 | 138,460 |
| 512K | 128,502 | 127,783 | 130,016 | 129,589 | 131,591 | 131,839 | 132,956 |
| 1024K | 127,797 | 127,592 | 128,450 | 129,906 | 128,856 | 132,099 | 130,440 |

Table C.10: Replacement-scheme results for the chess middle game (with time stamping, 7-ply searches).

| | TwoBig1 | TwoDeep | Big1 | BigAll | Deep | New | Old |
|---|---|---|---|---|---|---|---|
| 16K | 433,734 | 446,535 | 487,496 | 501,011 | 513,109 | 588,646 | 632,460 |
| 64K | 363,502 | 370,538 | 397,446 | 395,755 | 411,775 | 494,620 | 501,081 |
| 256K | 319,972 | 330,656 | 357,510 | 341,741 | 343,183 | 382,383 | 397,500 |
| 1024K | 316,183 | 309,192 | 327,089 | 324,889 | 323,494 | 344,971 | 339,173 |

Table C.11: Replacement-scheme results for the chess middle game (without time stamping, 8-ply searches).

| | TwoBig1 | TwoDeep | Big1 | BigAll | Deep | New | Old |
|---|---|---|---|---|---|---|---|
| 16K | 421,898 | 441,366 | 495,898 | 493,721 | 527,186 | 590,553 | 706,031 |
| 64K | 354,250 | 366,945 | 400,034 | 393,498 | 415,413 | 493,801 | 545,271 |
| 256K | 321,320 | 321,361 | 342,764 | 337,188 | 349,470 | 397,421 | 407,303 |
| 1024K | 300,430 | 312,459 | 320,724 | 316,789 | 317,324 | 329,933 | 333,470 |

Table C.12: Replacement-scheme results for the chess middle game (with time stamping, 8-ply searches).

Second, the 10-ply results for the seven replacement schemes (TwoBig1, TwoDeep, Big1, BigAll, Deep, New and Old) on chess endgame positions are listed in Table C.13 with eight table sizes (8K, 16K, 32K, 64K, 128K, 256K, 512K and 1024K). All figures reported are number of nodes searched in millions. These results are the cumulative results of all five games (112 endgame positions) given in Appendix B. For every endgame experiment time stamping was used.

| | TwoBig1 | TwoDeep | Big1 | BigAll | Deep | New | Old |
|---|---|---|---|---|---|---|---|
| 8K | 608 | 638 | 909 | 997 | 953 | 1,034 | 1,569 |
| 16K | 508 | 533 | 740 | 779 | 758 | 914 | 1,392 |
| 32K | 434 | 447 | 615 | 632 | 599 | 807 | 1,124 |
| 64K | 396 | 407 | 517 | 520 | 501 | 698 | 916 |
| 128K | 365 | 359 | 428 | 415 | 408 | 607 | 692 |
| 256K | 334 | 330 | 372 | 367 | 388 | 528 | 505 |
| 512K | 309 | 304 | 333 | 327 | 334 | 460 | 396 |
| 1024K | 287 | 292 | 305 | 302 | 301 | 404 | 330 |

Table C.13: Replacement-scheme results for the chess endgame (with time stamping, 10-ply searches).

Third, the results in the domain of domineering for five replacement schemes (TwoBig1, TwoDeep, Big1, Deep and New) are listed with four table sizes (256K, 512K, 1024K and 2048K) in Table C.14. All figures reported are number of nodes searched in millions. The results are given for the empty standard (8×8) board. Obviously, no time stamping was used, since the test set consists of only one position.

### Quantifying the merits of move and score

For the second series of experiments the following six experimental searches have been performed.

1. Search without a transposition table.
2. Search with a traditional transposition table, without *score*.
3. Search with a traditional transposition table, without *move*.
4. Search with a traditional transposition table, without *move*, only storing and using the score information if the score is a *true value*.
5. Search with a traditional transposition table, without *move*, only storing and using the score information if the score is a *bound value*.
6. Search with a traditional transposition table, with *move* and *score*, storing and using the score information both if the score is a *true value* or a *bound value* (i.e., use the transposition table fully).

| | TwoBig1 | TwoDeep | Big1 | Deep | New |
|---|---|---|---|---|---|
| 256K | 1,212 | 1,298 | 1,659 | 1,930 | 3,742 |
| 512K | 885 | 939 | 1,122 | 1,283 | 2,743 |
| 1024K | 607 | 635 | 745 | 812 | 2,130 |
| 2048K | 442 | 452 | 492 | 504 | 1,380 |

Table C.14: Replacement-scheme results for domineering.

First, the 8-ply transposition-table results for the replacement scheme TwoBig1 on 18 consecutive WTM middle-game positions taken from the game Kasparov-Short, Amsterdam (round 2) 1994 (cf. Appendix A) are listed in Table C.15.

Second, the 10-ply results for the replacement scheme TwoBig1 on 21 consecutive WTM endgame positions taken from the game Rabinovich-Romanovsky, Leningrad 1934 (cf. Appendix B) are listed in Table C.16. The experiments have been performed with six table sizes (8K, 16K, 32K, 64K, 128K and 256K). For every experiment time stamping was used. All figures reported are number of nodes searched in thousands.

| | No tt | Tt-move | Tt-score | True tt-score | Bound tt-score | Traditional tt |
|---|---|---|---|---|---|---|
| 8K | 610,696 | 473,041 | 326,139 | 599,491 | 321,141 | 297,244 |
| 16K | 610,696 | 456,754 | 298,797 | 599,491 | 282,651 | 262,409 |
| 32K | 610,696 | 433,439 | 274,639 | 599,491 | 264,205 | 235,358 |
| 64K | 610,696 | 414,718 | 266,698 | 599,491 | 250,568 | 213,422 |
| 128K | 610,696 | 403,509 | 258,279 | 599,491 | 239,020 | 200,483 |
| 256K | 610,696 | 392,076 | 248,917 | 599,491 | 227,815 | 193,644 |

Table C.15: Transposition-table results for the chess middle game (with time stamping, 8-ply searches).

### Using additional memory

For the third series of experiments we have tested the results of storing an $n$-ply PV ($n = 2..5$) in an entry versus storing only the best move (a 1-ply PV).

First, the 8-ply transposition-table results for the replacement scheme TwoBig1 on 18 consecutive WTM middle-game positions taken from the game Kasparov-Short, Amsterdam (round 2) 1994 (cf. Appendix A) are listed in Table C.17.

Second, the 10-ply results for the replacement scheme TwoBig1 on 21 consecutive WTM endgame positions taken from the game Rabinovich-Romanovsky, Leningrad 1934 (cf. Appendix B) are listed in Table C.18. The experiments have

| | No tt | Tt-move | Tt-score | True tt-score | Bound tt-score | Traditional tt |
|---|---|---|---|---|---|---|
| 8K | 409,119 | 208,056 | 118,658 | 404,658 | 114,804 | 73,690 |
| 16K | 409,119 | 188,754 | 102,370 | 404,394 | 102,988 | 61,779 |
| 32K | 409,119 | 174,397 | 95,666 | 404,415 | 90,221 | 52,473 |
| 64K | 409,119 | 161,670 | 88,890 | 402,261 | 85,470 | 47,928 |
| 128K | 409,119 | 151,189 | 81,801 | 402,644 | 81,811 | 43,107 |
| 256K | 409,119 | 144,681 | 76,388 | 402,644 | 81,501 | 43,168 |

Table C.16: Transposition-table results for the chess endgame (with time stamping, 10-ply searches).

been performed with six table sizes (8K, 16K, 32K, 64K, 128K and 256K). For every experiment time stamping was used. All figures reported are number of nodes searched in thousands.

| | 1-ply PV | 2-ply PV | 3-ply PV | 4-ply PV | 5-ply PV |
|---|---|---|---|---|---|
| 8K | 297,244 | 295,662 | 294,646 | 295,330 | 292,025 |
| 16K | 262,409 | 259,041 | 261,433 | 261,788 | 258,357 |
| 32K | 235,358 | 231,825 | 239,193 | 241,828 | 236,579 |
| 64K | 213,422 | 221,658 | 219,221 | 215,449 | 213,998 |
| 128K | 200,483 | 201,704 | 207,532 | 202,358 | 203,506 |
| 256K | 193,644 | 194,858 | 188,259 | 187,287 | 184,820 |

Table C.17: PV results for the chess middle game (with time stamping, 8-ply searches).

| | 1-ply PV | 2-ply PV | 3-ply PV | 4-ply PV | 5-ply PV |
|---|---|---|---|---|---|
| 8K | 73,690 | 75,732 | 77,254 | 74,340 | 73,209 |
| 16K | 61,779 | 63,815 | 62,303 | 60,851 | 62,768 |
| 32K | 52,473 | 52,432 | 51,213 | 55,629 | 51,898 |
| 64K | 47,928 | 44,365 | 50,903 | 46,377 | 43,603 |
| 128K | 43,107 | 41,250 | 43,680 | 41,919 | 42,128 |
| 256K | 43,168 | 37,208 | 38,843 | 37,451 | 37,744 |

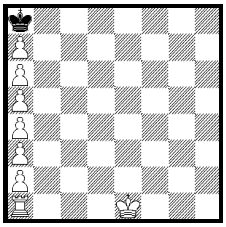
Table C.18: PV results for the chess endgame (with time stamping, 10-ply searches).

# Appendix D

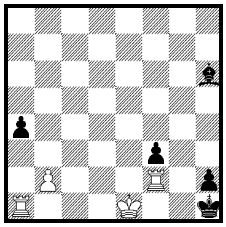
# The pn-search and pn$^2$-search test set

This appendix lists the test set of 117 positions used for the proof-number-search experiments described in Chapters 3, 4, and 5.

The following WTM positions from Reinfeld (1958) and Krabbé (1985) have been used:



Krabbé #35



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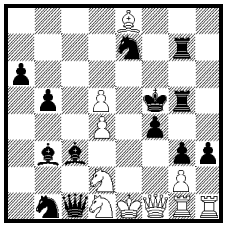
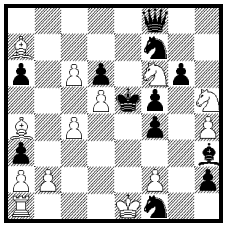
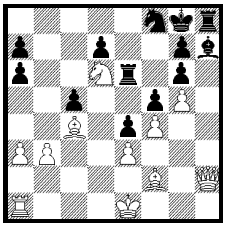


Krabbé #38



Krabbé #40



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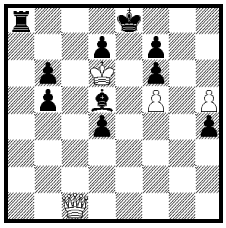
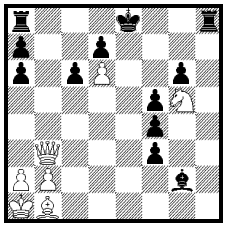
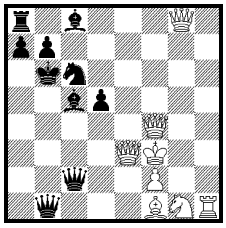


Krabbé #60



Krabbé #61



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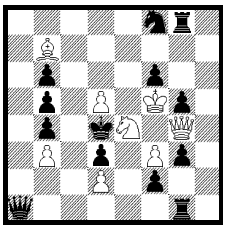
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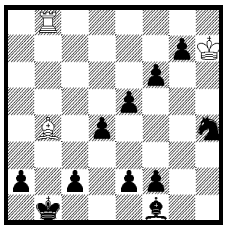
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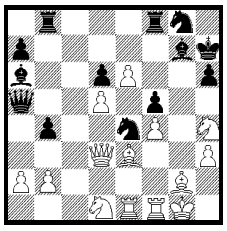
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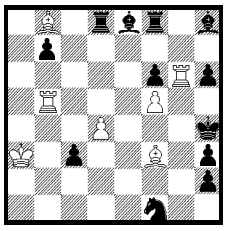
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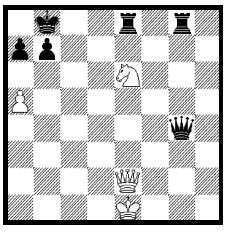
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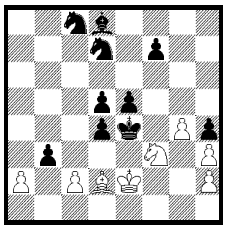
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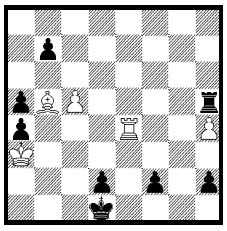
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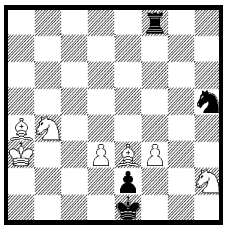
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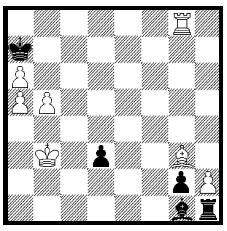
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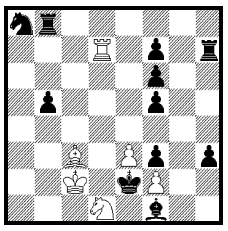
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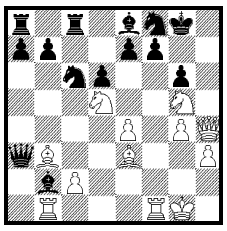
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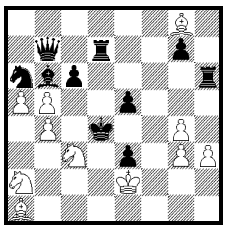
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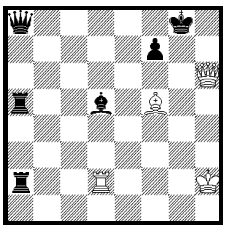
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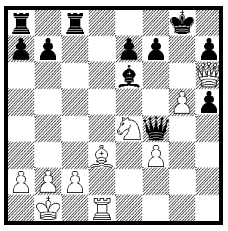
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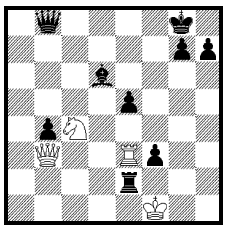
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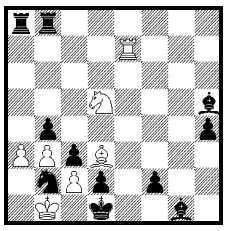
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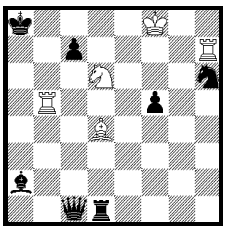
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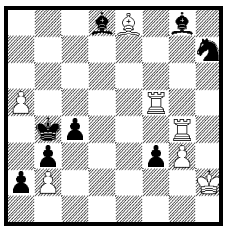
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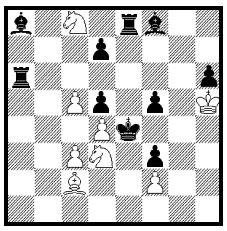
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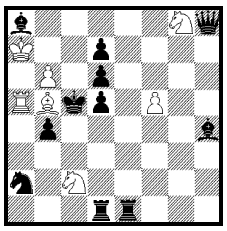
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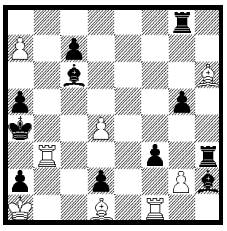
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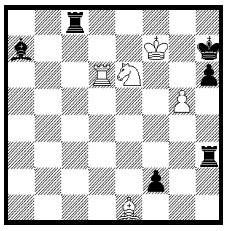
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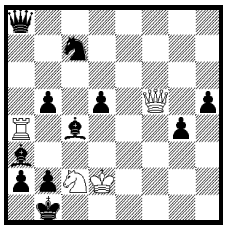
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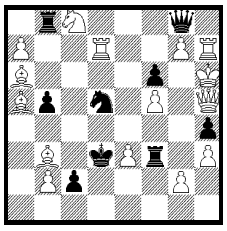
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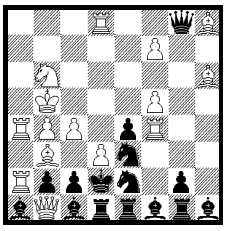
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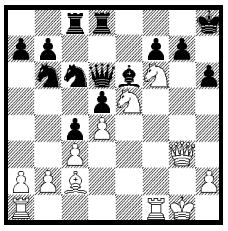
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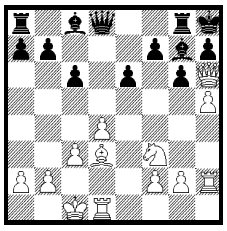
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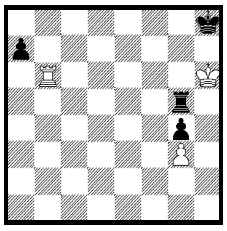
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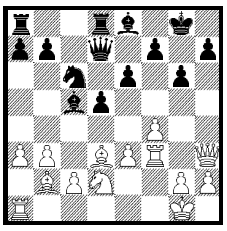
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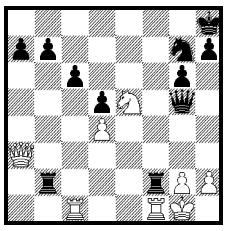
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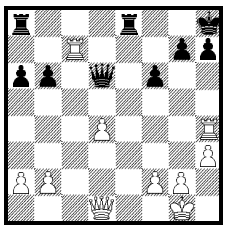
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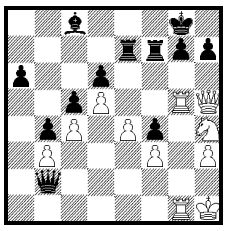
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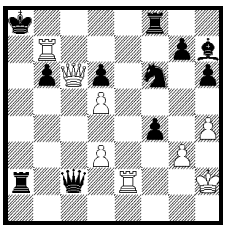
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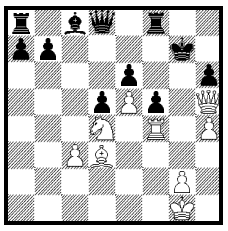
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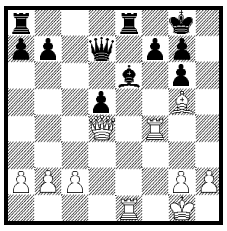
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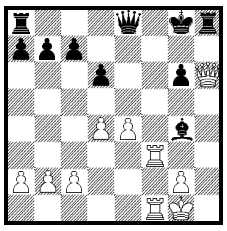
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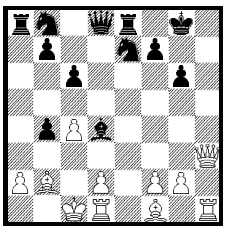
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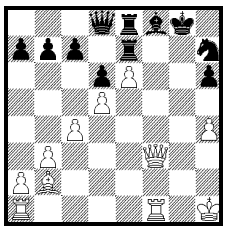
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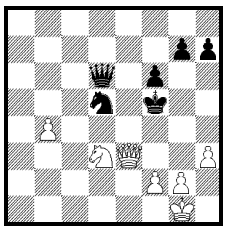
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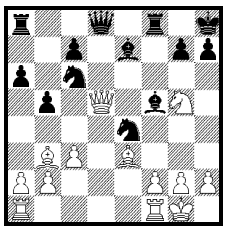
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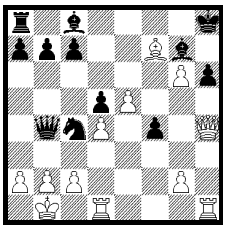
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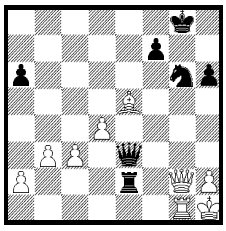
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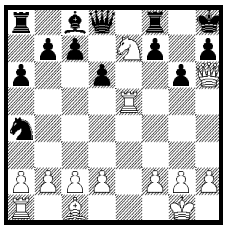
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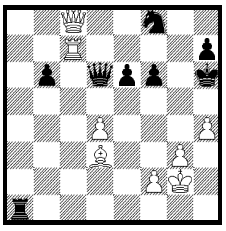
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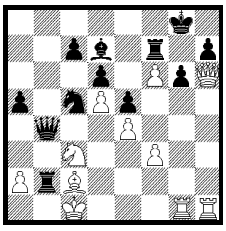
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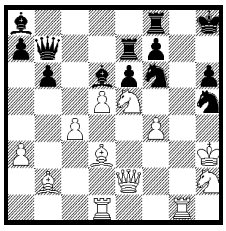
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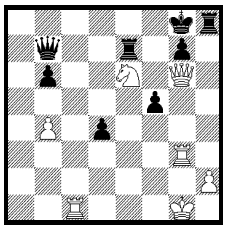
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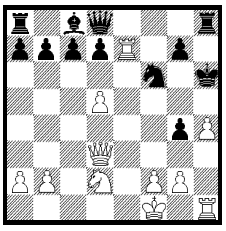
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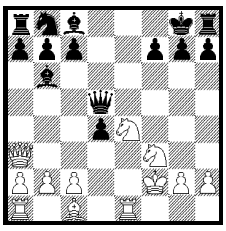
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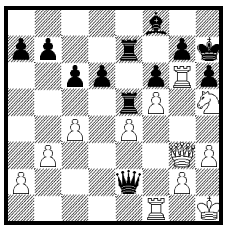
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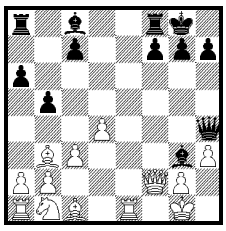
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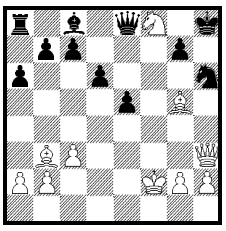
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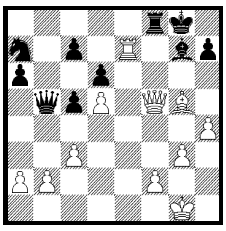
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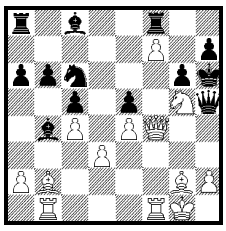
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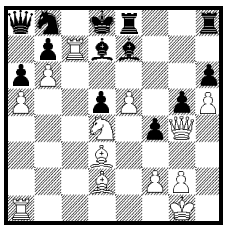
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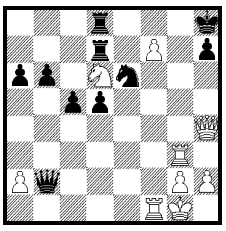
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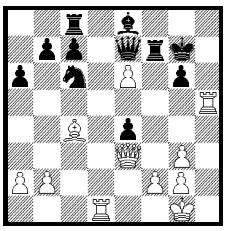
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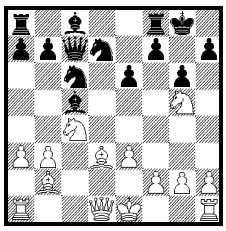
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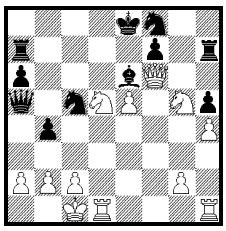
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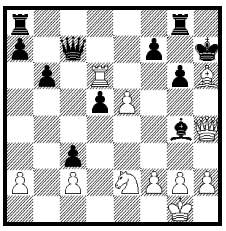
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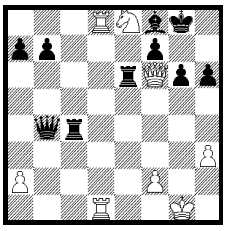
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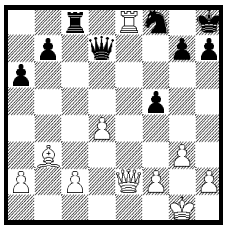
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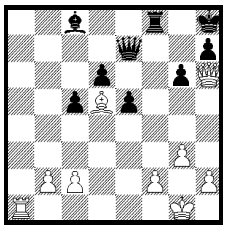
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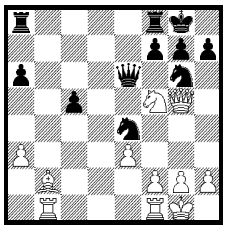
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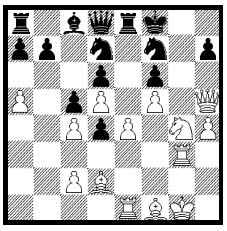
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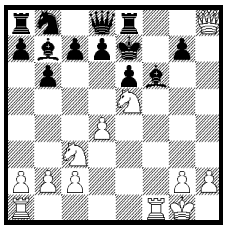
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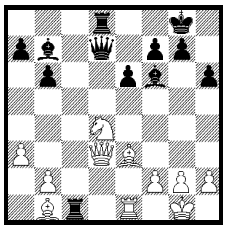
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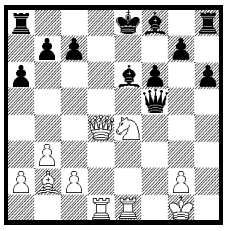
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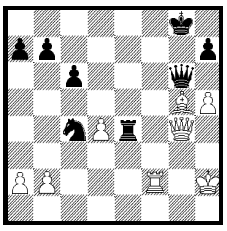
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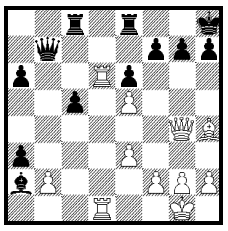
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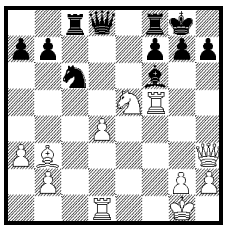
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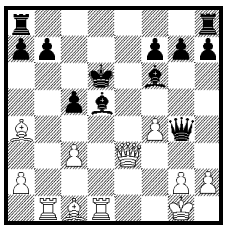
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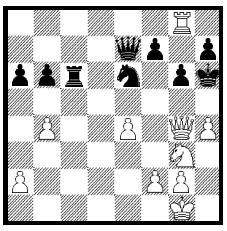
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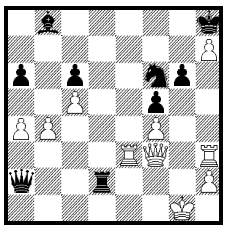
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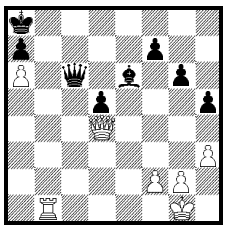
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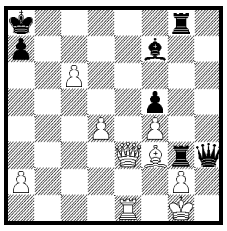
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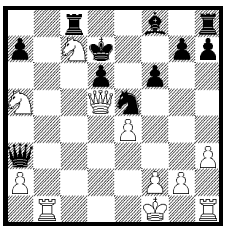
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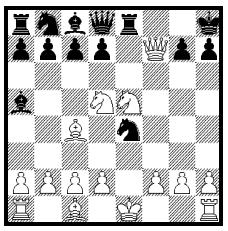
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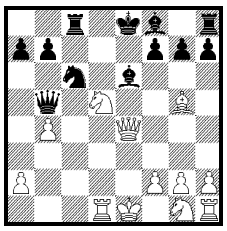
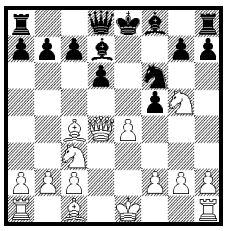
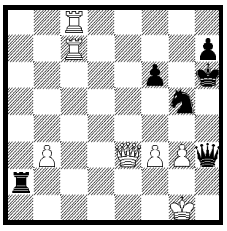
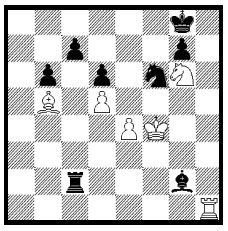
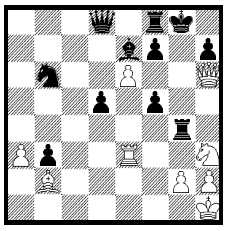
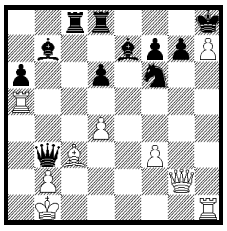

Reinfeld #267

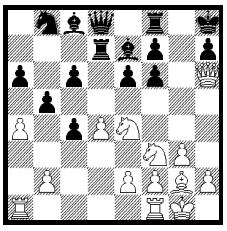

Reinfeld #278

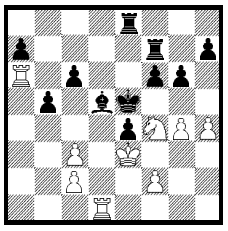

Reinfeld #281

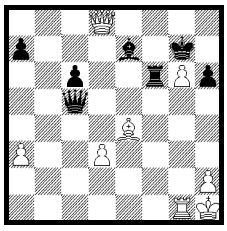

Reinfeld #282


Reinfeld #283

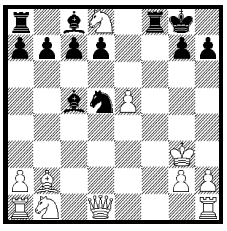

Reinfeld #285

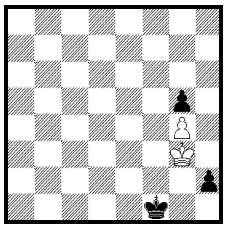

Reinfeld #293

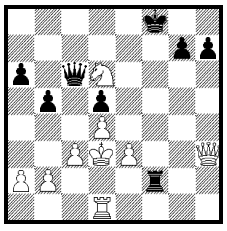

Reinfeld #295

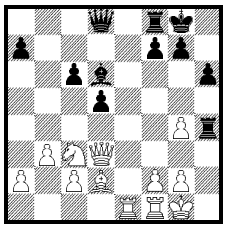

Reinfeld #298

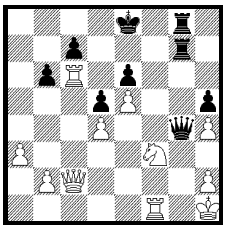
The following BTM positions from Reinfeld (1958) and Krabbé (1985) have been used:

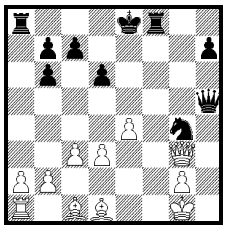

Krabbé #8

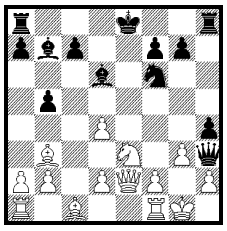

Krabbé #284

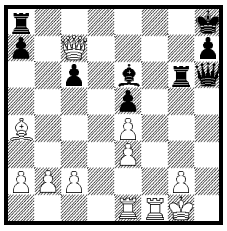

Reinfeld #5

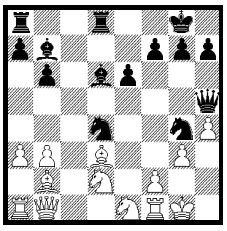

Reinfeld #9

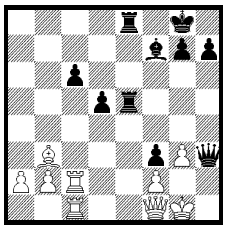

Reinfeld #12

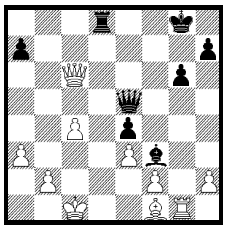

Reinfeld #54

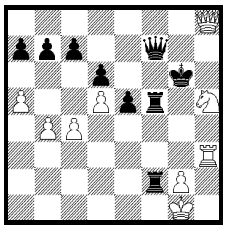

Reinfeld #79

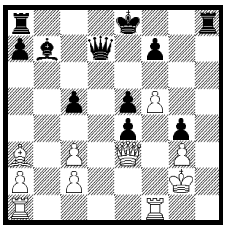

Reinfeld #88

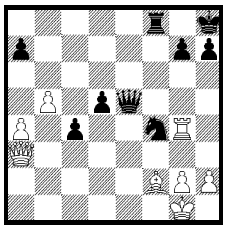

Reinfeld #105

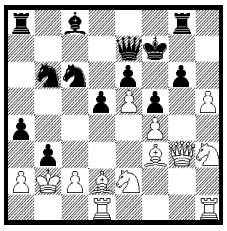

Reinfeld #132

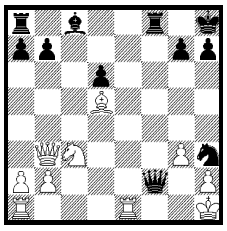

Reinfeld #134

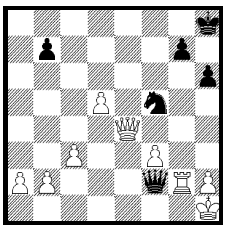

Reinfeld #167

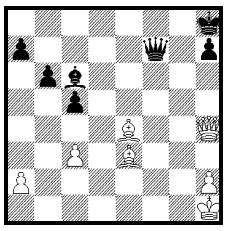

Reinfeld #168

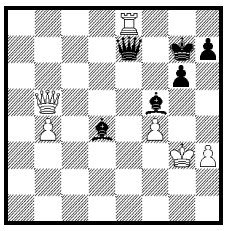

Reinfeld #172

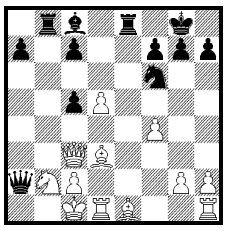

Reinfeld #177

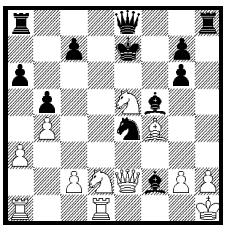

Reinfeld #179

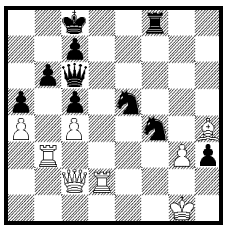

Reinfeld #197


Reinfeld #219


Reinfeld #225


Reinfeld #252


Reinfeld #266


Reinfeld #290

# Appendix E

# The pn-search versus $\alpha\beta$-search results

This appendix presents the results of the experiments with the pn-search algorithm and the $\alpha\beta$-seach algorithm described in Chapter 3. In Table E.1 all results are listed for the test set of 117 positions. The numbers refer to the number of nodes searched. A dash signifies that no solution is found due to the memory constraints (1,000,000 nodes). The first column lists the test positions. Columns two and three show the results for pn search and $\alpha\beta$ search, respectively.

| | Pn search | $\alpha\beta$ search |
|---|---|---|
| K8 | – | – |
| K35 | 296 | 244,122 |
| K37 | 43,221 | – |
| K38 | 273 | 57,509 |
| K40 | – | – |
| K44 | – | – |
| K60 | – | 930,899 |
| K61 | 42,228 | – |
| K78 | – | – |
| K192 | 23,290 | – |
| K194 | 229,423 | – |
| K195 | – | – |
| K196 | 298,428 | – |
| K197 | 323 | 15,831 |
| K198 | 247,435 | – |

| | Pn search | $\alpha\beta$ search |
|---|---|---|
| K199 | 370,016 | – |
| K206 | 15,978 | – |
| K207 | 95,418 | – |
| K208 | 62,791 | – |
| K209 | – | – |
| K210 | – | – |
| K211 | 957 | 155,641 |
| K212 | 81,842 | 104,368 |
| K214 | 685 | – |
| K215 | 114,060 | – |
| K216 | 592,890 | – |
| K217 | – | – |
| K218 | 118,361 | – |
| K219 | 310,447 | – |
| K220 | – | – |

Table E.1: Comparing pn search and $\alpha\beta$ search (continued on next page).

| | Pn search | αβ search |
|---|---|---|
| K261 | 482 | 906 |
| K284 | – | 224,092 |
| K317 | 173,480 | 158,523 |
| K333 | 145,922 | – |
| K334 | 217,516 | – |
| R1 | 7,640 | 18,225 |
| R4 | 82 | 52 |
| R5 | 57 | 22 |
| R6 | 71,966 | – |
| R9 | 207 | 124,234 |
| R12 | 175 | 1,174 |
| R14 | 324,542 | 127,519 |
| R27 | 77 | 270 |
| R35 | 527 | 3,421 |
| R49 | 16,546 | 545,344 |
| R50 | 183 | 830 |
| R51 | 227,361 | – |
| R54 | 85 | 1,631 |
| R55 | 31,456 | 393,646 |
| R57 | 113 | 106 |
| R60 | 69 | 107 |
| R61 | 78 | 537 |
| R64 | 137 | 1,201 |
| R79 | 152 | 49,502 |
| R84 | 93 | 609 |
| R88 | 759 | 21,862 |
| R96 | – | – |
| R97 | 107 | 7,186 |
| R99 | 75,411 | 12,227 |
| R102 | 279 | 208 |
| R103 | 2,150 | 20,662 |
| R104 | 5,047 | 14,017 |
| R105 | – | 777,182 |
| R132 | 2,301 | 6,340 |
| R134 | 854 | 19,008 |
| R136 | 185 | 114 |
| R138 | 211,466 | – |
| R139 | 274 | 47,199 |
| R143 | 900 | 13,023 |
| R154 | 117 | 1,784 |
| R156 | 82 | 1,081 |
| R158 | 526 | 5,416 |
| R159 | 385,487 | – |
| R160 | 110 | 2,783 |

| | Pn search | αβ search |
|---|---|---|
| R161 | 2,045 | 4,212 |
| R167 | 896 | 178,495 |
| R168 | 596,956 | – |
| R172 | 99 | 6,062 |
| R173 | 419 | 5,729 |
| R177 | 527 | 5,538 |
| R179 | 184 | 10,342 |
| R182 | 807,709 | – |
| R184 | 82 | 1,026 |
| R186 | 108 | 30,588 |
| R188 | 117 | 94 |
| R191 | 22,466 | 17,952 |
| R197 | 95 | 762 |
| R201 | – | – |
| R203 | 19,917 | 6,166 |
| R211 | 278 | 282,863 |
| R212 | 458 | 204,009 |
| R215 | 164 | 3,244 |
| R217 | 271 | 331,404 |
| R218 | 277,639 | – |
| R219 | 157 | 414 |
| R222 | 59,591 | – |
| R225 | 342 | 3,034 |
| R241 | 360,983 | – |
| R244 | 458 | 70,466 |
| R246 | 120 | 753 |
| R250 | 1,147 | – |
| R251 | 136,479 | 384,761 |
| R252 | 537,628 | – |
| R253 | 2,355 | 311 |
| R260 | 807 | 511,553 |
| R263 | 887 | 45,180 |
| R266 | 716 | 289,510 |
| R267 | 1,206 | 22,890 |
| R278 | 636 | 389,195 |
| R281 | 317,214 | – |
| R282 | 749 | 58,274 |
| R283 | 30,778 | 102,501 |
| R285 | 218 | 2,372 |
| R290 | 523 | 42,650 |
| R293 | 121,720 | – |
| R295 | 81 | 3,584 |
| R298 | 150 | 3,900 |

Table E.1: Comparing pn search and αβ search (continued).

# Appendix F

# The pn$^2$-search results

This appendix lists the results of the experiments with the pn$^2$-search algorithm and its modifications.

In Chapter 4 it is stated that when parameter $a$ becomes large and parameter $b$ becomes small the fraction function approaches $f(x) = 0$, which means that standard pn search is used. When both parameter $a$ and parameter $b$ have a small positive value the fraction function approaches $f(x) = 1$, which means that the pn$^2$-search algorithm suggested by Allis (1994) is used. Table F.1 confirms these observations. In the first column of the table the algorithm is given. The second column states the number of positions solved, and the third column states the total number of nodes searched. We note that the pn-search result differs from the result with ($a$ $b$) equal to (999K 1), because the first result stems from the immediate-evaluation variant of pn search, and the second from the delayed-evaluation variant of pn search.

| Algorithm | Solved | # nodes |
|---|---|---|
| Pn search | 92 | 2,974,602 |
| $a$=999K $b$=1 | 87 | 2,392,664 |
| $a$=1 $b$=1 | 108 | 67,085,784 |
| Pn$^2$ search (Allis) | 108 | 67,085,784 |

Table F.1: Two extremes of the fraction function.

Next, all results of the experiments described in Chapter 4 are presented in Table F.2. For these experiments, the test set consists of 108 positions. In the first two columns the values of parameters $a$ and $b$ are given. The third column states the number of positions solved. The sizes of the first-level and second-level tree are listed in columns four and five, respectively. Column six shows the total number of nodes searched over the solved positions. Finally, the maximum number of nodes in memory for the most difficult test position is given in column seven.

| $a$ | $b$ | # | First level | Second level | Total | Maximum |
|---|---|---|---|---|---|---|
| 75K | 3,750 | 108 | 2,668,522 | 414,715,977 | 417,384,499 | 122,822 |
| 75K | 7.5K | 108 | 2,107,363 | 183,059,544 | 185,166,907 | 97,880 |
| 75K | 15K | 108 | 1,321,283 | 58,630,011 | 59,951,294 | 70,808 |
| 75K | 30K | 108 | 825,798 | 56,792,248 | 57,618,046 | 48,488 |
| 75K | 37.5K | 108 | 737,686 | 55,092,677 | 55,830,363 | 42,945 |
| 75K | 45K | 108 | 658,312 | 52,527,548 | 53,185,860 | 40,428 |
| 75K | 60K | 108 | 599,335 | 54,678,255 | 55,277,590 | 35,580 |
| 75K | 75K | 108 | 553,540 | 55,622,054 | 56,175,594 | 39,947 |
| 75K | 90K | 108 | 538,282 | 57,474,549 | 58,012,831 | 39,984 |
| 75K | 120K | 108 | 510,672 | 58,163,405 | 58,674,077 | 37,322 |
| 75K | 150K | 108 | 503,728 | 59,589,317 | 60,093,045 | 36,568 |
| 75K | 180K | 108 | 494,585 | 59,457,269 | 59,951,854 | 36,563 |
| 75K | 210K | 108 | 488,951 | 59,792,279 | 60,281,230 | 36,003 |
| 75K | 240K | 108 | 492,391 | 61,418,574 | 61,910,965 | 35,605 |
| 100K | 10K | 108 | 2,482,057 | 141,392,048 | 143,874,105 | 110,920 |
| 100K | 60K | 108 | 663,291 | 50,661,253 | 51,324,544 | 46,305 |
| 100K | 90K | 108 | 561,438 | 49,790,831 | 50,352,269 | 34,653 |
| 125K | 12.5K | 108 | 2,818,440 | 104,339,512 | 107,157,952 | 120,979 |
| 125K | 75K | 108 | 659,378 | 46,992,565 | 47,651,943 | 43,770 |
| 125K | 90K | 108 | 621,220 | 49,335,318 | 49,956,538 | 38,737 |
| 150K | 3,750 | 108 | 4,998,281 | 612,351,208 | 617,349,489 | 189,770 |
| 150K | 7.5K | 108 | 4,390,473 | 269,286,304 | 273,676,777 | 166,447 |
| 150K | 15K | 108 | 3,113,015 | 80,038,932 | 83,151,947 | 135,643 |
| 150K | 30K | 108 | 1,728,353 | 60,406,646 | 62,134,999 | 91,605 |
| 150K | 60K | 108 | 901,278 | 54,777,822 | 55,679,100 | 55,430 |
| 150K | 75K | 108 | 793,386 | 56,532,843 | 57,326,229 | 53,954 |
| 150K | 90K | 108 | 674,730 | 49,173,880 | 49,848,610 | 48,665 |
| 150K | 120K | 108 | 601,153 | 49,506,906 | 50,108,059 | 37,630 |
| 150K | 150K | 108 | 555,561 | 50,273,586 | 50,829,147 | 34,647 |
| 150K | 180K | 108 | 526,613 | 51,573,591 | 52,100,204 | 33,413 |
| 150K | 210K | 108 | 518,422 | 54,791,716 | 55,310,138 | 38,152 |
| 150K | 240K | 108 | 510,498 | 56,215,959 | 56,726,457 | 38,249 |
| 150K | 480K | 108 | 498,061 | 63,168,302 | 63,666,363 | 31,185 |
| 200K | 20K | 108 | 3,642,974 | 58,523,263 | 62,166,237 | 164,472 |
| 200K | 90K | 108 | 853,735 | 54,127,970 | 54,981,705 | 55,158 |
| 200K | 120K | 108 | 696,652 | 50,890,364 | 51,587,016 | 48,860 |
| 250K | 25K | 108 | 4,046,147 | 46,244,259 | 50,290,406 | 175,617 |
| 250K | 90K | 108 | 1,057,131 | 55,266,557 | 56,323,688 | 68,215 |
| 250K | 150K | 108 | 697,125 | 49,876,992 | 50,574,117 | 49,780 |
| 300K | 15K | 108 | 6,904,133 | 85,195,061 | 92,099,194 | 275,206 |
| 300K | 22.5K | 108 | 5,558,725 | 43,618,137 | 49,176,862 | 237,670 |
| 300K | 30K | 108 | 4,391,781 | 43,512,164 | 47,903,945 | 200,110 |

Table F.2: The pn² results for varying parameters $a$ and $b$
(continued on next page).

| $a$ | $b$ | # | First level | Second level | Total | Maximum |
|---|---|---|---|---|---|---|
| 300K | 40K | 108 | 3,219,230 | 39,278,406 | 42,497,636 | 162,991 |
| 300K | 60K | 108 | 2,003,287 | 46,485,285 | 48,488,572 | 117,496 |
| 300K | 90K | 108 | 1,248,374 | 48,739,998 | 49,988,372 | 77,854 |
| 300K | 120K | 108 | 957,952 | 53,597,265 | 54,555,217 | 59,768 |
| 300K | 150K | 108 | 817,824 | 56,359,721 | 57,177,545 | 55,957 |
| 300K | 165K | 108 | 753,545 | 51,515,985 | 52,269,530 | 55,570 |
| 300K | 180K | 108 | 711,403 | 51,190,005 | 51,901,408 | 51,969 |
| 300K | 210K | 108 | 647,192 | 49,930,739 | 50,577,931 | 44,686 |
| 300K | 240K | 108 | 618,720 | 51,738,723 | 52,357,443 | 40,312 |
| 300K | 300K | 108 | 553,488 | 48,820,125 | 49,373,613 | 36,234 |
| 300K | 480K | 108 | 516,863 | 56,576,815 | 57,093,678 | 37,855 |
| 450K | 15K | 88 | 2,685,315 | 77,055 | 2,762,370 | >300,000 |
| 450K | 22.5K | 98 | 5,309,701 | 2,889,136 | 8,198,837 | >300,000 |
| 450K | 30K | 104 | 6,159,818 | 13,468,281 | 19,628,099 | >300,000 |
| 450K | 45K | 108 | 5,200,437 | 31,899,341 | 37,099,778 | 277,633 |
| 450K | 60K | 108 | 3,710,957 | 32,101,854 | 35,812,811 | 200,822 |
| 450K | 75K | 108 | 2,755,338 | 37,285,569 | 40,040,907 | 158,558 |
| 450K | 90K | 108 | 2,158,171 | 43,057,915 | 45,216,086 | 135,091 |
| 450K | 120K | 108 | 1,458,198 | 45,147,453 | 46,605,651 | 98,068 |
| 450K | 150K | 108 | 1,173,016 | 50,363,222 | 51,536,238 | 74,863 |
| 450K | 180K | 108 | 989,985 | 55,097,838 | 56,087,823 | 65,040 |
| 450K | 210K | 108 | 850,180 | 52,617,722 | 53,467,902 | 55,486 |
| 450K | 240K | 108 | 772,110 | 50,392,270 | 51,164,380 | 50,924 |
| 450K | 270K | 108 | 720,221 | 51,678,441 | 52,398,662 | 52,916 |
| 450K | 300K | 108 | 655,616 | 47,560,738 | 48,216,354 | 45,910 |
| 450K | 330K | 108 | 634,798 | 48,572,363 | 49,207,161 | 40,442 |
| 450K | 360K | 108 | 614,915 | 49,918,022 | 50,532,937 | 40,806 |
| 450K | 450K | 108 | 560,286 | 49,245,643 | 49,805,929 | 36,882 |
| 600K | 15K | 87 | 2,392,664 | 0 | 2,392,664 | >300,000 |
| 600K | 30K | 91 | 3,539,437 | 433,025 | 3,972,462 | >300,000 |
| 600K | 60K | 106 | 5,175,810 | 21,486,835 | 26,662,645 | >300,000 |
| **600K** | **80K** | **108** | **4,061,359** | **30,818,703** | **34,880,062** | **236,383** |
| 600K | 90K | 108 | 3,429,832 | 32,596,751 | 36,026,583 | 200,682 |
| 600K | 120K | 108 | 2,198,328 | 37,433,618 | 39,631,946 | 136,878 |
| 600K | 150K | 108 | 1,632,654 | 42,467,933 | 44,100,587 | 112,288 |
| 600K | 180K | 108 | 1,310,916 | 46,726,924 | 48,037,840 | 87,436 |
| 600K | 210K | 108 | 1,137,297 | 51,167,545 | 52,304,842 | 73,973 |
| 600K | 240K | 108 | 1,006,380 | 55,098,476 | 56,104,856 | 68,357 |
| 600K | 300K | 108 | 826,932 | 55,119,547 | 55,946,479 | 59,813 |
| 600K | 360K | 108 | 719,206 | 49,798,501 | 50,517,707 | 47,901 |
| 600K | 480K | 108 | 614,823 | 49,558,639 | 50,173,462 | 39,913 |
| 600K | 600K | 108 | 558,575 | 48,982,121 | 49,540,696 | 36,773 |

Table F.2: The $\text{pn}^2$ results for varying parameters $a$ and $b$
(continued on next page).

| $a$ | $b$ | # | First level | Second level | Total | Maximum |
|---|---|---|---|---|---|---|
| 750K | 15K | 87 | 2,392,664 | 0 | 2,392,664 | >300,000 |
| 750K | 30K | 87 | 2,392,664 | 0 | 2,392,664 | >300,000 |
| 750K | 37.5K | 87 | 2,392,664 | 834 | 2,393,498 | >300,000 |
| 750K | 60K | 99 | 4,668,848 | 4,396,135 | 9,064,983 | >300,000 |
| 750K | 75K | 104 | 4,879,717 | 12,059,792 | 16,939,509 | >300,000 |
| 750K | 90K | 107 | 4,625,718 | 24,204,635 | 28,830,353 | >300,000 |
| 750K | 120K | 108 | 3,226,125 | 32,090,839 | 35,316,964 | 206,505 |
| 750K | 150K | 108 | 2,296,757 | 39,959,428 | 42,256,185 | 149,546 |
| 750K | 180K | 108 | 1,731,845 | 39,282,953 | 41,014,798 | 116,815 |
| 750K | 210K | 108 | 1,447,288 | 46,590,740 | 48,038,028 | 98,804 |
| 750K | 240K | 108 | 1,269,027 | 51,306,410 | 52,575,437 | 83,348 |
| 750K | 300K | 108 | 1,015,055 | 55,693,427 | 56,708,482 | 68,297 |
| 750K | 450K | 108 | 720,103 | 49,808,809 | 50,528,912 | 48,265 |
| 750K | 600K | 108 | 614,273 | 49,255,159 | 49,869,432 | 39,933 |
| 750K | 750K | 108 | 564,262 | 49,982,423 | 50,546,685 | 36,757 |

Table F.2: The pn² results for varying parameters $a$ and $b$ (continued).

# Appendix G

# The BTA results for pn search

This appendix presents the results of the experiments with the pn-search algorithm and its modifications described in Chapter 5. In Table G.1 the results of the 117 test positions are listed for four pn-search variants with the same move ordering. The numbers refer to the number of nodes searched. A dash signifies that no solution was found due to the memory constraints (500,000 nodes). The first column lists the test positions. Columns two to five show the results for the tree algorithm[1], the DAG algorithm, the DCG algorithm, and the BTA algorithm, respectively.

| | *Tree* | *DAG* | *DCG* | *BTA* |
|---|---|---|---|---|
| K8 | – | – | – | – |
| K35 | 296 | 296 | 276 | 276 |
| K37 | 35,724 | 25,737 | 17,981 | 19,886 |
| K38 | 273 | 273 | 272 | 272 |
| K40 | – | – | – | – |
| K44 | – | 274,211 | 274,211 | 146,938 |
| K60 | – | 310,251 | 372,634 | 487,969 |
| K61 | 43,911 | 41,997 | 35,770 | 38,446 |
| K78 | – | – | – | – |
| K192 | 22,525 | 15,429 | 14,252 | 15,767 |
| K194 | 238,085 | 51,427 | 30,699 | 102,336 |

Table G.1: The results for four pn-search variants (continued on next page).

[1] The numbers differ from the numbers given in Appendix E, because there the tree algorithm uses a different move ordering.

| | *Tree* | *DAG* | *DCG* | *BTA* |
|---|---|---|---|---|
| K195 | – | – | – | – |
| K196 | 318,276 | 97,717 | 88,069 | 93,447 |
| K197 | 429 | 429 | 417 | 413 |
| K198 | 333,165 | 262,255 | 171,720 | 177,929 |
| K199 | 369,555 | 290,903 | 151,043 | 202,903 |
| K206 | 11,931 | 11,543 | 9,483 | 9,191 |
| K207 | 236,568 | 88,024 | 41,348 | 50,870 |
| K208 | 72,468 | 65,279 | – | 31,648 |
| K209 | – | – | – | – |
| K210 | – | – | – | – |
| K211 | 1,059 | 1,059 | 939 | 937 |
| K212 | 83,413 | 59,988 | 52,946 | 55,290 |
| K214 | 645 | 645 | 629 | 624 |
| K215 | 124,984 | 94,108 | – | 74,967 |
| K216 | – | 366,336 | – | 247,686 |
| K217 | – | – | 311,027 | 407,633 |
| K218 | 122,058 | 109,308 | 124,868 | 107,215 |
| K219 | 277,250 | 129,232 | 63,297 | 83,329 |
| K220 | – | – | – | – |
| K261 | 414 | 388 | 388 | 424 |
| K284 | – | 2,337 | 2,337 | 2,851 |
| K317 | 157,424 | 120,358 | 103,033 | 94,043 |
| K333 | 165,725 | 134,339 | 123,599 | 139,184 |
| K334 | 145,291 | 88,430 | 74,375 | 82,889 |
| R1 | 4,275 | 4,095 | 3,996 | 4,270 |
| R4 | 82 | 82 | 82 | 82 |
| R5 | 57 | 57 | 57 | 57 |
| R6 | 96,059 | 32,953 | 11,703 | 13,179 |
| R9 | 173 | 173 | 169 | 168 |
| R12 | 99 | 99 | 99 | 99 |
| R14 | 335,936 | 213,098 | 157,269 | 185,918 |
| R27 | 77 | 77 | 77 | 77 |
| R35 | 597 | 559 | 371 | 522 |
| R49 | 16,935 | 14,767 | 13,797 | 15,625 |
| R50 | 399 | 383 | 369 | 408 |
| R51 | 270,495 | 191,822 | 173,922 | 204,299 |
| R54 | 256 | 256 | 256 | 256 |
| R55 | 15,245 | 13,293 | 12,749 | 14,552 |
| R57 | 287 | 287 | 287 | 287 |
| R60 | 69 | 69 | 69 | 69 |
| R61 | 78 | 78 | 78 | 78 |
| R64 | 153 | 153 | 153 | 153 |

Table G.1: The results for four pn-search variants
(continued on next page).

| | *Tree* | *DAG* | *DCG* | *BTA* |
|---|---|---|---|---|
| R79 | 152 | 152 | 152 | 152 |
| R84 | 93 | 93 | 93 | 93 |
| R88 | 595 | 547 | 542 | 583 |
| R96 | – | – | – | – |
| R97 | 107 | 107 | 107 | 107 |
| R99 | 31,767 | 31,302 | 27,264 | 27,290 |
| R102 | 199 | 199 | 199 | 199 |
| R103 | 1,837 | 1,742 | 1,731 | 1,780 |
| R104 | 5,042 | 4,660 | 4,658 | 4,870 |
| R105 | – | – | – | – |
| R132 | 2,291 | 2,105 | 2,077 | 2,135 |
| R134 | 804 | 798 | 758 | 760 |
| R136 | 230 | 230 | 230 | 230 |
| R138 | 192,886 | 164,106 | 118,729 | 137,030 |
| R139 | 182 | 182 | 182 | 182 |
| R143 | 521 | 520 | 520 | 519 |
| R154 | 197 | 197 | 196 | 196 |
| R156 | 82 | 82 | 82 | 82 |
| R158 | 495 | 495 | 494 | 494 |
| R159 | 403,797 | 253,108 | 274,275 | 263,275 |
| R160 | 110 | 110 | 110 | 110 |
| R161 | 1,790 | 1,209 | 1,209 | 1,332 |
| R167 | 923 | 901 | 813 | 810 |
| R168 | – | 317,557 | 209,725 | 301,527 |
| R172 | 99 | 99 | 99 | 99 |
| R173 | 419 | 418 | 404 | 402 |
| R177 | 349 | 349 | 349 | 349 |
| R179 | 156 | 156 | 156 | 156 |
| R182 | – | 230,648 | 212,087 | 372,096 |
| R184 | 82 | 82 | 82 | 82 |
| R186 | 108 | 108 | 108 | 108 |
| R188 | 117 | 117 | 117 | 117 |
| R191 | 22,830 | 20,480 | 17,858 | 17,046 |
| R197 | 95 | 95 | 95 | 95 |
| R201 | – | – | – | – |
| R203 | 20,980 | 18,429 | 17,265 | 17,397 |
| R211 | 278 | 272 | 231 | 230 |
| R212 | 545 | 545 | 543 | 543 |
| R215 | 164 | 164 | 164 | 164 |
| R217 | 199 | 199 | 199 | 199 |
| R218 | 270,277 | 225,638 | 160,720 | 210,311 |
| R219 | 140 | 140 | 140 | 140 |

Table G.1: The results for four pn-search variants
(continued on next page).

| | *Tree* | *DAG* | *DCG* | *BTA* |
|---|---|---|---|---|
| R222 | 60,855 | 48,209 | 22,827 | 49,934 |
| R225 | 263 | 263 | 263 | 263 |
| R241 | 365,495 | 254,998 | 195,577 | 231,746 |
| R244 | 323 | 323 | 323 | 323 |
| R246 | 61 | 61 | 61 | 61 |
| R250 | 1,102 | 1,101 | 1,076 | 1,071 |
| R251 | 88,547 | 70,104 | 53,285 | 57,798 |
| R252 | – | – | 352,315 | 386,046 |
| R253 | 2,709 | 1,189 | 1,176 | 2,477 |
| R260 | 841 | 794 | 729 | 804 |
| R263 | 654 | 621 | 621 | 651 |
| R266 | 716 | 716 | 711 | 711 |
| R267 | 1,136 | 1,001 | 1,001 | 1,089 |
| R278 | 333 | 333 | 333 | 333 |
| R281 | 316,252 | 93,578 | – | 46,033 |
| R282 | 749 | 729 | 725 | 742 |
| R283 | 14,787 | 14,530 | 14,070 | 14,165 |
| R285 | 218 | 218 | 218 | 218 |
| R290 | 408 | 408 | 408 | 408 |
| R293 | 97,666 | 94,138 | 75,283 | 75,509 |
| R295 | 134 | 134 | 134 | 134 |
| R298 | 150 | 150 | 150 | 150 |

Table G.1: The results for four pn-search variants (continued).